

# A MATHEMATICAL ANALYSIS OF PNEUMATIC DRYING OF GRAINS—I. CONSTANT DRYING RATE

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**Abstract**—A mathematical model is developed for the pneumatic drying process. Important variables such as gas temperature and humidity, solids loadings, gas and solid velocities and consequently the heat and mass transfer coefficients are incorporated into the analysis. The study on the coupling behaviors as well as the effects of parameters provides many design criteria applicable to an actual drying process.

## NOMENCLATURE

$a_v$	solid surface area per unit dryer volume [ $\text{m}^{-1}$ ]
$C$	specific heat at constant pressure [ $\text{J kg}^{-1} \text{K}^{-1}$ ]
$C_d$	drag coefficient
$D$	diameter of dryer duct [m]
$d_p$	diameter of grain [m]
$Fr_0$	Froude number, $u_{a,0}/\sqrt{(gD)}$
$G$	mass flow rate (dry basis) [ $\text{kg s}^{-1}$ ]
$H$	absolute humidity
$h_p$	heat transfer coefficient around particle [ $\text{W m}^{-2} \text{K}^{-1}$ ]
$i$	enthalpy of unit mass [ $\text{J kg}^{-1}$ ]
$K_0, K_1$	parameters, equation (21)
$k_H$	mass transfer coefficient around particle [ $\text{kg m}^{-2} \text{s}^{-1}$ ]
$M$	molecular weight [kg]
$m$	mass flow ratio of dry solids to air, $G_s/G_a$
$Nu$	Nusselt number
$Pr$	Prandtl number
$q_i$	net heat flux into the solids [ $\text{W m}^{-2}$ ]
$R_d$	drying rate on the solid surface [ $\text{kg m}^{-2} \text{s}^{-1}$ ]
$Re$	Reynolds number based on the diameter of the duct
$Re_p$	Reynolds number around a single particle
$S$	cross sectional area of duct [ $\text{m}^2$ ]
$Sc$	Schmidt number
$Sh$	Sherwood number
$T$	temperature [K]
$t$	time [s]
$U$	dimensionless velocity, equation (13)
$u$	velocity [ $\text{m s}^{-1}$ ]
$V_p$	volume of single grain [ $\text{m}^3$ ]
$W$	moisture content (dry basis) [(kg-water) (kg-dry solid) $^{-1}$ ]
$X$	dimensionless distance, $x/D$
$x$	distance [m].

$\rho$	density [ $\text{kg m}^{-3}$ ]
$\psi_\mu$	relative viscosity of air, equation (11).

## Subscripts

a	dry air
f	final state
g	gas, or at gas temperature
H	humid
$\text{H}_2\text{O}$	water
m	moist
s	solid, or at solid temperature
sat	saturated
t	terminal velocity
v	water vapor
w	water or wet-bulb
0	initial state.

## 1. INTRODUCTION

PNEUMATIC drying, where wet materials are simply allowed to be conveyed vertically or horizontally in a pipe by hot air and hence dried simultaneously, has been proven practical commercially for many years. Advantages of this technique are: (a) large effective surface area of the material to be dried since solids are well dispersed within the dryer; (b) longer constant rate period compared with other bed-type drying systems since the resistance for liquid migration is reduced due to the smaller characteristic path within the solids; (c) less possibility of degeneration and charring of material even with higher temperatures of drying gas due to co-current flow and shorter residence time; (d) high drying capacity due to a continuous process. On the other hand, it is not suitable for highly adhesive or fragile materials.

Kamei and Toei [1] and Kamei *et al.* [2] first reported detailed experimental investigations on the mechanism of heat and mass transfer in pneumatic drying. Materials such as copper sulfate, activated carbon, sawdust, ammonium sulfate and sand were dried in a vertical steel pipe 0.1 m in diameter and 14.5 m in length. It was noted that the heat transfer coefficient is at a maximum in the acceleration region just behind the inlet section. The reason for such high coefficients, however, were not given. Recently, Debrand [3] experimentally determined the particle velocities and

## Greek symbols

$\lambda$	latent heat of vapor [ $\text{J kg}^{-1}$ ]
$\mu$	viscosity [ $\text{kg m}^{-1} \text{s}^{-1}$ ]

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flow patterns in a pneumatic flash dryer. It was indicated that there were three locations in which a high heat flux was observed between the gas and the particles; namely (a) the feeding point, (b) the elbows, and (c) the cyclone. The high flux at these locations was explained by high values of the particle slip velocities and high intensities of the turbulence phenomena.

There have also been published over the last few years a few theoretical approaches to the design of pneumatic dryers [4–7]. Nevertheless, it is difficult to find in the literature any precise simulation model of pneumatic dryers which takes account of both the velocity and the physical properties of the drying gas as well as elementary transfer phenomena between gas and particles, and design criteria on a pneumatic dryer.

In pneumatic drying processes, a progressive temperature drop along the path causes a considerable decrease in the gas velocity resulting from an appreciable change in the gas properties such as density and viscosity. An increase in humidity of the drying gas along the length of the dryer also causes a change in density. Since all these changes have definite influences upon a particle's motion, the situation becomes more complicated. Furthermore, any variation of particle velocity would affect the drying rate since heat and mass transfer coefficients to and from the solid particle is closely related to the relative velocity between the gas and the solids. Therefore, one must take these coupling effects into consideration in order to provide design criteria on a pneumatic dryer.

This paper proposes a mathematical model to simulate pneumatic drying which includes all these coupling effects between the particle motion and the drying mechanism.

## 2. MODEL DESCRIPTION

Assumptions which are common for pneumatic drying are:

(1) Gas temperatures and humidity concentrations are uniform over the cross section of the duct, but vary with the length of the duct.

(2) Solid particles are distributed uniformly in the cross section of the duct.

(3) Solid particles are spherical and of a uniform size.

(4) The duct wall is thermally insulated and there is no radiant energy exchange between the gas–solid suspension and the surroundings.

(5) The drying rate of the solid is constant hence it does not depend on the moisture content of the solids. (Falling rate drying will be examined in the following paper.)

(6) Gas and solids are flowing co-currently upward in suspension.

Material balance for the moisture content of solids may be expressed as

$$\frac{dW}{dx} = -(a_v S/G_s)R_d = (G_a/G_s)\frac{dH}{dx}, \quad (1)$$

where  $R_d$  is defined as  $k_H(H_{sa1} - H)$  and  $dH/dx$  is the humidity change along the duct. When a temperature difference between the gas and the solid is large, one should also take into account the enthalpy balance for both phases. Let  $q_i$  be a heat flux to the solid surface, the heat balance for the boundary film surrounding particle gives

$$h_p(T_g - T_s) + R_d i_{v,s} = q_i + R_d i_{v,g}, \quad (2)$$

where  $i_{v,s}$  and  $i_{v,g}$  are the enthalpy of vapor at  $T_s$  and  $T_g$ , respectively. The net heat flux into the solid is then

$$q_i = (T_g - T_s)(h_p - C_v R_d). \quad (3)$$

The enthalpy balance for the gas phase is

$$\frac{di_g}{dx} = [R_d i_{v,g} - h_p(T_g - T_s)](a_v S/G_a), \quad (4)$$

where  $i_g$  is a humid enthalpy per unit dry air. Elimination of  $i_g$  and  $i_{v,g}$  leads to

$$C_H \frac{dT_g}{dx} = -h_p(a_v S/G_a)(T_g - T_s). \quad (5)$$

In a similar way, the enthalpy balance for the solid phase is described as

$$(C_s + WC_w) \frac{dT_s}{dx} = \{h_p(T_g - T_s) - [C_v(T_g - T_s) + \lambda_s]R_d\}(a_v S/G_s), \quad (6)$$

where  $\lambda_s$  is the latent heat at the solid temperature  $T_s$ .

Since particles are transported upward by air flow, its motion is described by the following equation

$$\rho_{s,m} V_p \frac{du_s}{dt} = \frac{1}{2}(\pi d_p^2/4)\rho_g C_D(u_g - u_s)^2 - V_p \rho_{s,m}(1 - \rho_g/\rho_{s,m})g. \quad (7)$$

If one assumes  $\rho_g/\rho_{s,m} \ll 1$ , it becomes

$$\frac{du_s}{dt} = \frac{3}{4} \frac{\rho_g}{d_p \rho_s(1+W)} C_D(u_g - u_s)^2 - g, \quad (8)$$

where  $C_D$  is the drag coefficient which is dependent on particle Reynolds number

$$C_D = \begin{cases} 24/Re_p, & Re_p \leq 5 \text{ (Stokes' law)}, \\ 10/\sqrt{Re_p}, & 5 < Re_p < 500 \text{ (Allen's law)}, \\ 0.44, & 500 \leq Re_p \text{ (Newton's law)}. \end{cases} \quad (9)$$

On utilizing the relation  $u_s = dx/dt$

$$u_s \frac{du_s}{dx} = \frac{3}{4} \frac{\rho_g}{d_p \rho_s(1+W)} C_D(u_g - u_s)^2 - g, \quad (10)$$

where both  $\rho_g$  and  $u_g$  are functions of  $H$  and  $T_g$ . Arnold's correlation [8] is used here for describing the variation of air viscosity with temperature

$$\frac{\mu_{a,2}}{\mu_{a,1}} = \psi_\mu = \left(\frac{T_{g,2}}{T_{g,1}}\right)^{3/2} \left(\frac{T_{g,1} + 1.47T_b}{T_{g,2} + 1.47T_b}\right), \quad (11)$$

where  $T_b$  is the normal boiling temperature.

Heat and mass transfer coefficients to and from the solid particles are calculated by using the following well-known empirical equations [9]

$$Sh(or\ Nu) = 2 + 0.6Re_p^{1/2}Sc^{1/3}(or\ Pr^{1/3}). \tag{12}$$

Introducing the following dimensionless variables  $X = x/D, \ U_g = u_g/u_{a,0}, \ and \ U_s = u_s/u_{a,0}, \tag{13}$

the system equations are rearranged in non-dimensional forms as

$$\frac{dW}{dX} = -K_0(Sh/U_s)\psi_\mu(H_{sat} - H), \tag{14}$$

$$\frac{dH}{dX} = -m\frac{dW}{dX}, \tag{15}$$

$$[1 + (C_v/C_a)H]\frac{dT_g}{dX} = -K_0m(Sc/Pr) \times (Nu/U_s)\psi_\mu(T_g - T_s), \tag{16}$$

$$[(C_s/C_w) + W]\frac{dT_s}{dX} = -\frac{1}{m}[1 + (C_v/C_a)H] \times (C_a/C_w)\frac{dT_g}{dX} - \frac{1}{m}[(C_v/C_w)(T_g - T_s) + (\lambda_s/C_w)]\frac{dH}{dX}, \tag{17}$$

$$U_s\frac{dU_s}{dX} = K_1[\psi_{\rho,H}/(1 + W)]C_D(U_g - U_s)^2 - Fr_0^{-2}, \tag{18}$$

where  $\psi_{\rho,H}$  is a relative gas density

$$\psi_{\rho,H} = \frac{\rho_g}{\rho_{a,0}} = \left(\frac{T_{g,0}}{T_g}\right)\left(\frac{1 + H}{1 + H(M_a/M_{H_2O})}\right), \tag{19}$$

and the relative gas velocity  $U_g$  is expressed as a function of  $T_g$  and  $H$

$$U_g = (T_g/T_{g,0})[1 + H(M_a/M_{H_2O})]. \tag{20}$$

In summary, the parameters which control the system are  $K_0, K_1, m$  (mass flow ratio of solids to gas) and  $Fr_0$  (Froude number based on the superficial air velocity).  $K_0$  and  $K_1$  are defined as

$$K_0 = 6(D/d_p)^2(\rho_{a,0}/\rho_s)Re_0^{-1}Sc^{-1}, \tag{21}$$
  
$$K_1 = (3/4)(D/d_p)(\rho_{a,0}/\rho_s).$$

Among them,  $K_0$  is the characteristic of the pneumatic drying process concerned with mass transfer from a solid. It is the drying rate per unit mass of solid. It can be seen that the pneumatic drying is more suitable for materials having smaller sizes and lower densities.

Under the following boundary conditions at  $X = 0$   
 $H = H_0, \ T_g = T_{g,0}, \ T_s = T_{s,0},$

$$W = W_0 \quad and \quad U_s = 0,$$

the differential equations were solved by the Runge–Kutta–Gill method in which a relative error of each variable was controlled so as to be within 0.01% at any

Table 1. Size and density of grains used in the simulation

Materials	Size (mm)	Density (kg m <sup>-3</sup> )
Rape seed	1.67	1080
Wheat	4.0	1200
Corn	8.0	1280

point. The solid temperature at the inlet,  $T_{s,0}$ , was assumed to be the wet-bulb temperature for the humidity and temperature of air at the inlet. Solid particles used in the simulation are cereal grains such as wheat, corn and rape seed, physical properties of which are listed in Table 1, from a viewpoint of the application to drying of agricultural products.

3. RESULTS OF SIMULATION

Typical results of simulation for wheat grains are shown in Figs. 1 and 2, where data other than the mass flow ratio are maintained constant. Gas temperature declines along the length of dryer since sensible heat of gas is utilized for drying. It drops much faster with a higher mass flow ratio. With  $m = 0.5$  (Fig. 5), the temperature at  $x/D = 400$  reduces to less than a third of the initial value. Hence, the situation is nearly in a saturated state at this point. Consequently, humidity of the gas increases along the length and at a much faster rate with a high mass flow ratio. A rapid increase in humidity near the inlet represents a high drying rate due to a high relative velocity between gas and solids. This supports the experimental findings of Kamie and Toei [1], Kamei *et al.* [2] and Debrand [3]. Since the gas velocity must be maintained at a value high enough for conveyance of solids, the initial gas velocity to be chosen must take into account this significant drop in gas velocity along the path.

The solid accelerates in a short distance, and reaches an almost constant velocity at a low mass flow ratio as shown in Fig. 1. However, with a higher mass flow ratio as shown in Fig. 2, solid velocity begins to decline after reaching a maximum at a relatively short distance. This

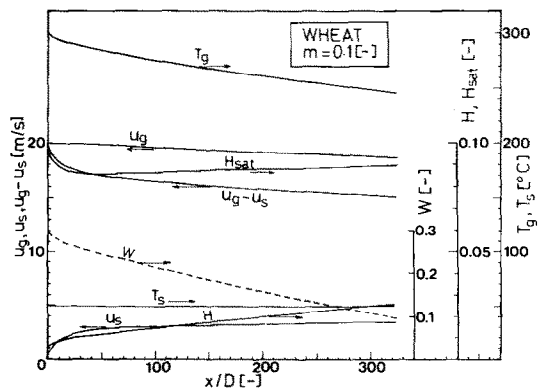


FIG. 1. Simulation result for wheat grains ( $m = 0.1, D/d_p = 20$ .  $D = 8 \times 10^{-2}$  m).

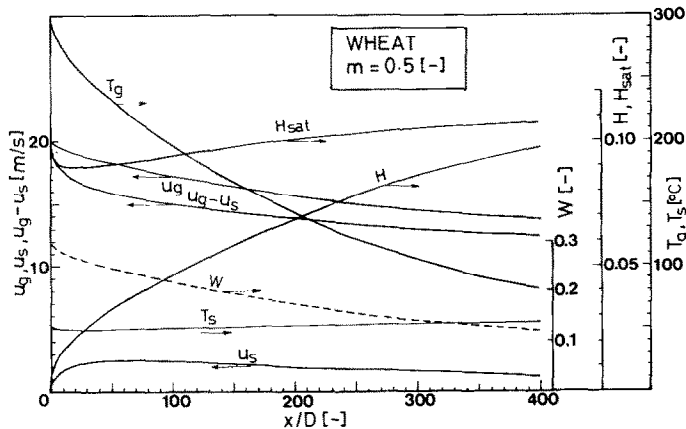


FIG. 2. Simulation result for wheat grains ( $m = 0.5$ ,  $D/d_p = 20$ ,  $D = 8 \times 10^{-2}$  m).

interesting phenomenon is caused by a significant drop in gas velocity due to decreasing temperatures. As can be seen from equation (18), solid velocity is dependent on humidity and temperature of the carrier gas as well as its own moisture content. Although a reduction of the moisture content and a humidity rise in the carrier gas tends to increase the solid velocity, the extensive drop in gas temperature is large enough to give a net result of decreasing the solid velocity as shown in Fig. 2.

Even after the flow of particles is fully developed, slip velocity, which is nearly equal to the terminal velocity of the solid, decreases accompanying gas velocity. Although the terminal velocity of solids decreases due to a reduction of density as drying progresses, the reduction of the gas velocity decreases at a much faster rate. Therefore, the slip velocity decreases along the path.

One of the main points of interest is how long a dryer is required to achieve a certain moisture content. It is found from the figures that a dryer longer than 30 m is required to dry wheat grains from a moisture content of 0.3–0.1 under those conditions. But to dry those from  $W = 0.3$  to 0.15, a dryer 20 m long should be required under the same conditions.

It can also be seen that the change in moisture content is highest just behind the inlet and thereafter nearly constant. The same is obtained in refs. [1–3]. In cases of higher mass flow ratio (Fig. 2), however, the drying rate shows a gradual decrease with distance. This contributed to a decrease in the driving force for mass and heat as gas temperature and humidity approach the saturated state.

The quantity  $a_v k_H$  is known as the capacity coefficient. A unit volume of the drying tower has a certain transfer capacity, depending on the magnitude of the capacity coefficient and of the driving force. The larger the capacity coefficient, the less the mass transfer resistance and consequently the smaller the volume of the dryer required per unit transfer capacity. In this sense, the capacity coefficient is a useful criterion in comparing column performance. However, what differs from other usual cases such as a packed or fluidized column is the fact that the surface area of solid per unit

column volume is not constant through the pneumatic dryer as the solid velocity changes along the path. Moreover, the mass transfer coefficient,  $k_H$ , is also affected by the solid velocity. The capacity coefficients obtained under conditions of Figs. 1 and 2 are shown in Fig. 3. High values of the coefficients just behind the inlet are due to a very small solid velocity. That is, as the particle is not yet accelerated fully in a short distance, numbers of particles in a unit volume of duct are so high and consequently gives the largest solid surface area per unit volume of duct. Moreover, a large slip velocity due to small solid velocity causes an increase in the mass transfer coefficient. As solids are accelerated, the capacity coefficient approaches a steady value. However, for higher mass flow ratios ( $m = 0.4$  and 0.5), the coefficient presents a trend of slight increase along the distance after taking a minimum value. This is caused by the fact that the solid velocity decreases slightly due to a gas temperature drop as mentioned above and therefore both  $a_v$  and  $k_H$  increase.

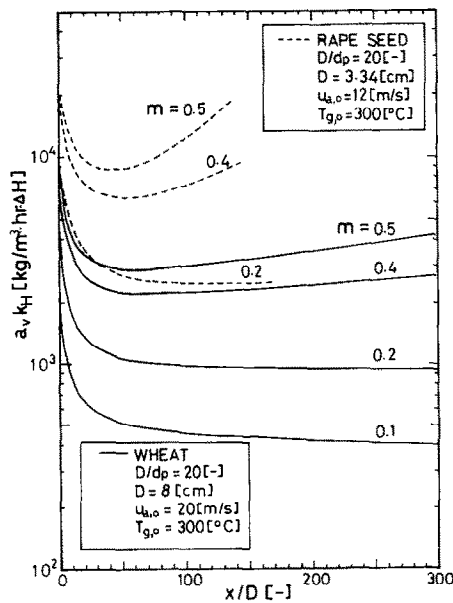


FIG. 3. Effect of mass flow ratio upon the capacity coefficient.

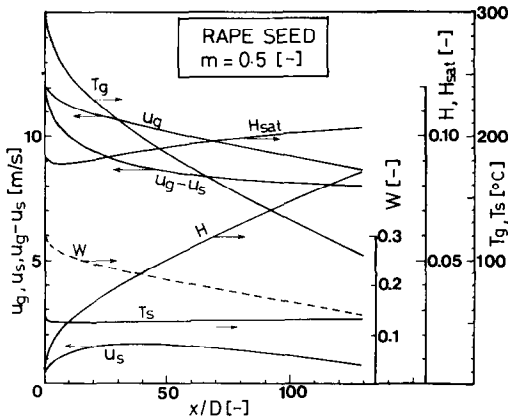


FIG. 4. Simulation result for rape seed grains ( $m = 0.5$ ,  $D/d_p = 20$ ,  $D = 3.34 \times 10^{-2}$  m).

#### 4. PARAMETER STUDIES

There are several parameters which influence the drying capacity of a pneumatic dryer required to reduce the product moisture content to a desired moisture level for a particular grain. These include gas temperature, mass flow ratio, gas flow rate, grain size, duct size and final moisture content. When one designs a pneumatic drying process, there are various limitations on these parameters.

##### 4.1. Grain size

An example of simulation for rape seed is shown in Fig. 4, where the gas temperature is of the same value as in the case of wheat (Figs. 1 and 2). In this case, as the grain size is less than half of that of wheat, the drying rate is greatly increased, and thus the dryer length required to reduce the same moisture content is shortened to almost a fifth of that for wheat grain. This is also reflected in the difference in the capacity coefficient, as shown in Fig. 3. The capacity coefficients of rape seed are greater than those of wheat, since both  $a_p$  and  $k_H$  increase with decreasing particle size. This is further illustrated in Fig. 5 for the drying of corn. In this case, 50 m is required.

##### 4.2. Gas temperature

The most important factor in selecting gas temperature is a maximum grain temperature allowed in the drying process. The gas temperature must be kept below the maximum value depending on the intended use of the grain. A maximum grain temperature of  $43^\circ\text{C}$  is usually recommended for seed ( $52^\circ\text{C}$  will kill the grain in most grain); for milling, a temperature above  $60^\circ\text{C}$  should be avoided; and for feed grains,  $88^\circ\text{C}$  is often considered as the limit [10]. These temperature limitations are difficult to apply precisely, because the temperature effect is coupled with the time of exposure to that temperature. A relatively high gas temperature can be used for pneumatic drying due to a short residence time. For a gas temperature of  $300^\circ\text{C}$  as used in the simulation, the maximum temperature is well below  $60^\circ\text{C}$ . Therefore, it can be considered as the optimum for pneumatic drying. This is much higher than the temperature allowed for other drying processes.

##### 4.3. Mass flow ratio

The mass flow ratio of solids to dry air is one of the important parameters especially for pneumatic drying. In the design of a pneumatic dryer, one would like to maximize the mass flow ratio from the viewpoint of reducing the equipment cost. There is, however, a limit. For a given initial and final moisture content of solids and the temperature and the humidity of the drying air at the inlet, the maximum mass flow ratio can be calculated by the following mass balance equation

$$m_{\max} = (H_i - H_0)/(W_0 - W_f). \quad (22)$$

A predicting method for the final humidity of the drying air,  $H_f$ , is presented in the Appendix. Figure 6 shows examples of the maximum mass flow ratio, calculated by equation (22), as a function of the initial gas temperature. It can be seen from the figure that the maximum mass flow ratio increases almost linearly with an increase in the gas temperature. If the mass flow ratio is chosen beyond a maximum value, the saturation should occur fast before the moisture content changes to a desired value.

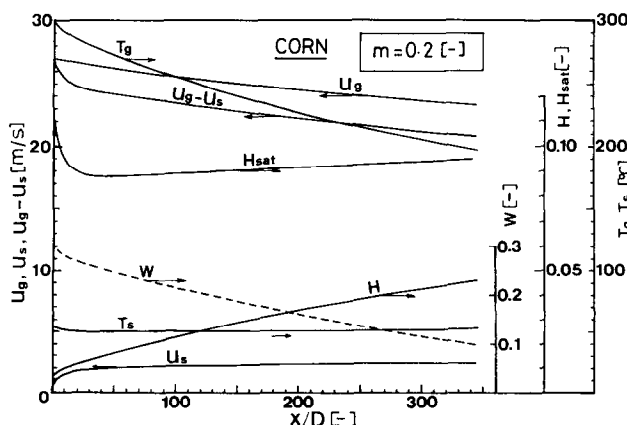


FIG. 5. Simulation result for corn grains ( $m = 0.2$ ,  $D/d_p = 20$ ,  $D = 16 \times 10^{-2}$  m).

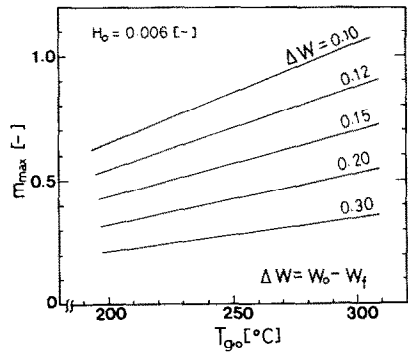


FIG. 6. Relationships between maximum mass flow ratio and initial gas temperature.

4.4. Gas flow rate

It is often difficult to select a gas flow rate, since gas velocity must be greater than the terminal velocity of the solid particle at any distance along the path in order to avoid choking in the dryer. The gas temperature decreases gradually along the path as the drying proceeds, leading to a decrease in gas velocity. On the other hand, an increase in the gas density due to a temperature drop and a decrease in the solid density as drying proceeds, reduce the terminal velocity of particles. Therefore, one must take into account both changes. The procedure of the determination of an optimum gas flow rate is shown in the Appendix. The optimum initial gas velocity should be

$$u_{a,0} = u_{t,f}(T_{g,0}/T_w)/[1 + (M_a/M_{H_2O})H_f]. \quad (23)$$

The relations between initial and final gas velocity are illustrated in Fig. 7 with parameters of gas temperature. The terminal velocities at the final state are also plotted for three species of solid. Intersections of the  $u_{t,f}$  line with the  $u_{g,f}$  line represent optimum values of the initial gas velocity.

5. CONCLUSIONS

Numerical simulations for pneumatic drying were performed on the basis of the mathematical model into

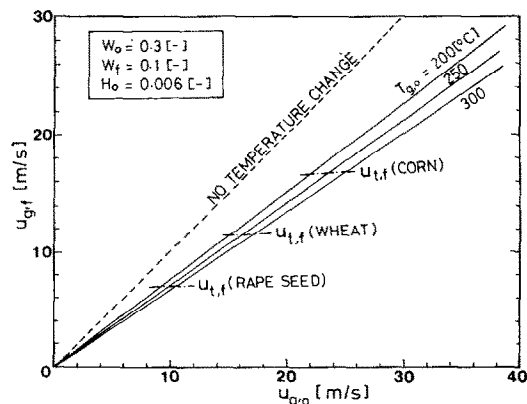


FIG. 7. Relationships between initial and final gas velocity.

which coupling effects among temperature of drying gas, gas properties, solid velocity and mass and heat transfer coefficients were considered. As a result, the following conclusions were obtained :

- (1) Advantages of pneumatic drying are distinctly exhibited for smaller grain sizes.
- (2) As the gas velocity decreases due to a drop in gas temperature, the choice of the initial value of the gas velocity is very important to ensure the stability of pneumatic transport.
- (3) A value of the mass flow ratio applicable is restricted by the gas temperature and the desired change of moisture content.

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APPENDIX

The solid temperature can be assumed almost constant throughout the path and nearly equal to the wet-bulb temperature for the initial condition, though, in the strict sense, it slightly changes along the path. The gas temperature at the exit is assumed equal to the solid temperature, and then to the wet-bulb temperature at the inlet,  $T_{g,f} = T_w$ .

Since the process is thermally insulated, the sum of the enthalpies of both phases must be maintained

$$\Delta i_g + m \Delta i_s = 0, \quad (A1)$$

where  $\Delta i_g$  and  $\Delta i_s$  are enthalpy changes of phases between the initial and final states, respectively. On supposing  $T_{s,f} = T_{s,0} = T_w$  and rearranging equation (A1), one has

$$C_{H,f}T_w - C_{H,0}T_{g,0} + \lambda_0(H_f - H_0) = m(W_0 - W_f)C_wT_w. \quad (A2)$$

Substituting equation (22) into equation (A2) and rewriting  $C_{H,f}$  with  $H_f$ , the final humidity becomes

$$H_f = H_0 + (T_{g,0} - T_w)C_{H,0}/\lambda_w, \quad (A3)$$

where  $\lambda_w$  is the latent heat at  $T_w$ . Substitution of equation (A3) into equation (22) gives the maximum mass flow ratio.

By using the above results, one can easily calculate the gas properties such as density and viscosity, and then the terminal velocity of particles at the exit condition. Finally, the relationship between the initial and final gas velocities,

expressed as

$$u_{g,f} = u_{a,0}(T_{g,f}/T_{g,0})[1 + H_f(M_a/M_{H_2O})], \quad (A4)$$

gives an optimum value of the initial gas velocity, equation (23).

#### UNE ANALYSE MATHÉMATIQUE DU SÉCHAGE PNEUMATIQUE DES GRAINS—I. VITESSE DE SÉCHAGE CONSTANTE

**Résumé**—On développe un modèle mathématique de mécanisme du séchage pneumatique. Les variables importantes telles que la température et l'humidité du gaz, les charges solides, les vitesses de gaz et de solide et par conséquent les coefficients de chaleur et de masse sont introduites dans l'analyse. L'étude des comportements de couplage et les effets des paramètres fournissent des critères de conception applicable à un processus industriel et de séchage.

#### MATHEMATISCHE UNTERSUCHUNG DER PNEUMATISCHEN TROCKNUNG VON GETREIDE—I. KONSTANTE TROCKNUNGSGESCHWINDIGKEIT

**Zusammenfassung**—Es wurde ein mathematisches Modell für den pneumatischen Trocknungsvorgang entwickelt. Die wichtigsten Variablen wie Lufttemperatur und -feuchtigkeit, Feststoffdurchsatz, Gas- und Feststoffgeschwindigkeit sowie die Wärme- und Stoffübergangskoeffizienten sind in der Berechnung berücksichtigt. Sowohl die Untersuchung des Kopplungsverhaltens als auch der Einzeleinflüsse der Parameter lieferte viele Auslegungskriterien, die sich auf einen wirklichen Trocknungsprozeß anwenden lassen.

#### МАТЕМАТИЧЕСКИЙ АНАЛИЗ ПНЕВМОГАЗОВОЙ СУШКИ ЗЕРНА— I. ПЕРИОД ПОСТОЯННОЙ СКОРОСТИ СУШКИ

**Аннотация**—Разработана математическая модель процесса пневмогазовой сушки. При анализе учитывается изменение температуры и влажности, массы твердой фазы, скоростей газовой и твердой фаз и, следовательно, коэффициентов тепло- и массопереноса. Проведенное исследование взаимного влияния различных параметров позволило получить целый ряд расчетных критериев, которые могут применяться на практике для описания процесса сушки.